NEUTRINO SPIN AND FLAVOUR CONVERSION AND OSCILLATIONS IN MAGNETIC FIELD

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Abstract

A review of the neutrino conversion and oscillations among the two neutrino species (active and sterile) induced by strong twisting magnetic field is presented and implications to neutrinos in neutron star, supernova, the Sun and interstellar galactic media are discussed. The "cross-boundary effect" (CBE) (i.e., a possible conversion of one half of neutrinos of the bunch from active into sterile specie) at the surface of neutron star is also studied for a realistic neutron star structure.

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1 Introduction

It is commonly believed that investigations of the neutrino properties will give a very important information for better understanding of particle interactions and for the progress in theoretical models. One of crucial problems of neutrino physics is the existance (or non existance) of the neutrino oscillations.

Neutrino oscillations if observed experimentally would play an important role in explanation of different astrophysical phenomena. The subsequent investigations in this field are strongly stimulated first of all by a possible solution of the solar-neutrino puzzle on the base of the matter and magnetic field enhancement of spin and flavour neutrino conversion (see, for example, [1]–[8] and [9]– [13] for a review). Another important motivation for consideration of neutrino conversion and oscillations is based on the common belief that these effects may be involved in the processes of supernova bursts and cooling of neutron stars (see [14]–[20] and references therein).

The basic idea of the neutrino conversion and oscillations between the two neutrino species in vacuum was put forward in [21] and was supplied [22, 23] with the time evolution analysis of a neutrino beam.

Effects of neutrino interaction with matter of uniform density on neutrino conversion were considered in [1].

The flavour conversion in the case of neutrino propagation in matter with nonuniform density was studied in [2, 3] and the resonant amplification of neutrino oscillations was predicted (MSW effect).

The neutrino spin precession in magnetic field as a possible solution for the solar-neutrino problem was studied in [4]–[6]. The resonant spin-flavour neutrino conversion that is analogous to the MSW effect was applied to the solar neutrino [7, 8] and also to neutrino from supernova [8].

It must be mentioned here that in the most of the performed studies of neutrino conversion and oscillations between two species in magnetized matter the considered strengths of magnetic field is of the order of $B \leq 10^5\ G$ that is quite adequate to the solar neutrino problem. There are also studies and discussions of the neutrino resonant conversion for the case of a supernova accounting for much stronger magnetic fields (see, for example, [8, 14, 19, 20]). However, in some recent studies in this field the possible influence of strong magnetic fields on neutrino conversion and oscillations was not considered at all (see, for example, [15]–[18],[24]).

The magnetic fields of the order of $B \sim 10^{12} - 10^{14}\ G$ are believed to exist at different stages of evolution of neutron stars. As an example we mention here new particle interaction phenomena [25, 26] that can be induced by strong magnetic fields and that can play a visible role in energetics of neutron

stars (see also [27]). The presence of strong magnetic fields may also influence the neutrino conversion and oscillations processes.

In this paper supposing that neutrinos have non-vanishing magnetic or/and flavour transition moments we study the magnetic field induced effects of neutrino spin and/or spin-flavour conversion and oscillations between different neutrino species. Both the Dirac and Majorana neutrino conversion and oscillations effects induced by strong magnetic fields in the presence of matter, also accounting for mixing of neutrinos in vacuum are considered.

We focus on the discussion of the case when under the influence of strong enough magnetic field numerous acts of conversion between the two neutrino specie occur (i. e., neutrino oscillations take place) for each individual neutrino of the bunch passing through different media. In Section 2 a general analysis of the problem is presented and the critical strength of magnetic field $\tilde{B}_{cr}$ as a function of characteristics of neutrino and matter is introduced (for magnetic fields $B \geq \tilde{B}_{cr}$ the magnetic field induced conversion and oscillation effects become important). The neutrino oscillations in magnetic field of a neutron star and the "cross-boundary effect" (CBE) [28]–[30] is discussed (Section 3). The CBE for a realistic neutron star structure accounting for variation of the matter density with distance from the centre of the star is studied in Section 4. In Section 5 the application of the magnetic field induced neutrino oscillations to the supernova reheating problem is discussed, and also neutrino oscillations in the galactic and twisting solar magnetic fields are considered.

2 General Analysis of Neutrino Oscillations in Magnetic Field

The evolution of neutrinos propagating in matter and transverse twisting magnetic field $\vec{B} = \vec{B}_{\perp} e^{i\phi(t)}$, (the angle $\phi(t)$ defines the direction of the field in the plane orthogonal to the neutrino momentum) is described by the Schrödinger-type equation

$$i\frac{d}{dt}\nu(t) = H\nu(t), \tag{1}$$

where the Hamiltonian H can be expressed as a sum of the four terms ([30]–[32])

$$H = H_V + H_{int} + H_F + H_{\dot{\phi}}. \tag{2}$$

Here H_V contains a contribution from a vacuum mass matrix, H_{int} contains a contribution from neutrino interactions with matter, H_F contains a contribution from interactions with the magnetic field and the last term $H_{\dot{\phi}}$ accounts for the effect of rotation (twisting) of the magnetic field.

If for the case of Dirac neutrinos one uses the bases in which neutrinos have a definite projection along the direction of propagation

$$\nu = (\nu_{e_L}, \nu_{\mu_L}, \nu_{e_R}, \nu_{\mu_R}), \tag{3}$$

then the Hamiltonian is given by (see [13],[30]– [34])

$$H^D = \begin{pmatrix} V^-_{\nu_e} & \frac{\Delta m^2_\nu}{4E_\nu}s & \mu_{ee}B & \mu_{e\mu}B \\ \frac{\Delta m^2_\nu}{4E_\nu}s & V^-_{\nu_\mu} & \mu_{\mu e}B & \mu_{\mu\mu}B \\ \mu_{ee}B & \mu_{\mu e}B & -\frac{\Delta m^2_\nu}{4E_\nu} + \frac{\dot\phi}{2} & 0 \\ \mu_{e\mu}B & \mu_{\mu\mu}B & 0 & \frac{\Delta m^2_\nu}{4E_\nu} + \frac{\dot\phi}{2} \end{pmatrix}. \tag{4}$$

The Hamiltonian (4) corresponds to the case of sterile neutrinos ν_{e_R} and ν_{μ_R}.

For the two Majorana neutrinos in the bases written as

$$\nu = (\nu_e, \nu_\mu, \bar\nu_e, \bar\nu_\mu)$$

in the corresponding Hamiltonian

$$H^M = \begin{pmatrix} V^-_{\nu_e} & \frac{\Delta m^2_\nu}{4E_\nu}s & 0 & \mu B \\ \frac{\Delta m^2_\nu}{4E_\nu}s & V^-_{\nu_\mu} & -\mu B & 0 \\ 0 & -\mu B & V^+_{\nu_e} & \frac{\Delta m^2_\nu}{4E_\nu}s \\ \mu B & 0 & \frac{\Delta m^2_\nu}{4E_\nu}s & V^+_{\nu_\mu} \end{pmatrix}, \tag{5}$$

where

$$V^-_{\nu_e} = -\frac{\Delta m^2_\nu}{4E_\nu}c + V^0_{\nu_e} - \frac{\dot\phi}{2},$$

$$V^-_{\nu_\mu} = \frac{\Delta m^2_\nu}{4E_\nu}c + V^0_{\nu_\mu} - \frac{\dot\phi}{2},$$

$$V^+_{\nu_e} = -\frac{\Delta m^2_\nu}{4E_\nu}c - V^0_{\nu_e} + \frac{\dot\phi}{2},$$

$$V^+_{\nu_\mu} = \frac{\Delta m^2_\nu}{4E_\nu}c - V^0_{\nu_\mu} + \frac{\dot\phi}{2}.$$

μ denotes the flavour transition magnetic moment.

Using these Hamiltonians we can consider different neutrino conversion processes $\nu_i \to \nu_j$ and the corresponding neutrino oscillations $\nu_i \leftrightarrow \nu_j$, induced by the magnetic field such as

$$\nu_{e_L} \to \nu_{e_R},\ \nu_{e_L} \to \nu_{\mu_R},\ \nu_{e_L} \to \bar\nu_{\mu_R}. \tag{6}$$

The probabilities of neutrino conversion from the type i (ν_i) to the type j (ν_j) after passing a distance x in matter and twisting magnetic field are

$$P(\nu_i \to \nu_j) = \sin^2 2\theta_{eff} \sin^2\Big(\frac{\pi x}{L_{eff}}\Big), i \neq j, \tag{7}$$

while the survival probabilities are

$$P(\nu_i \to \nu_i) = 1 - P(\nu_i \to \nu_j), \tag{8}$$

where the effective mixing angle θ_{eff} and effective oscillation length L_{eff} are given by

$$\tan 2\theta_{eff} = \frac{2\tilde{\mu}B}{\frac{\Delta m_\nu^2}{2E}A^\star - \sqrt{2}G_F n_{eff} + \dot{\phi}}, \tag{9}$$

$$L_{eff} = 2\pi\Big[\Big(\frac{\Delta m_\nu^2}{2E}A^\star - \sqrt{2}G_F n_{eff} + \dot{\phi}\Big)^2 + (2\tilde{\mu}B)^2\Big]^{-1/2}. \tag{10}$$

Note that the effective mixing angle θ_{eff} and effective oscillation length L_{eff} depend on the characteristics of the magnetic field rotation $\dot{\phi}$ along the neutrino pass (see also [24, 33, 34]).

For different neutrino conversion processes (6) $\tilde{\mu}$, $A^\star$ and n_{eff} are equal to

$$\tilde{\mu} = \begin{cases} \mu_{ee} & for\ \nu_{e_L} \to \nu_{e_R} \\ \mu_{e\mu} & for\ \nu_{e_L} \to \nu_{\mu_R} \\ \mu & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{11}$$

$$A^\star = \begin{cases} \frac{1}{2}(\cos 2\theta - 1) & for\ \nu_{e_L} \to \nu_{e_R} \\ \frac{1}{2}(\cos 2\theta + 1) & for\ \nu_{e_L} \to \nu_{\mu_R} \\ \cos 2\theta & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \end{cases}, \tag{12}$$

$$n_{eff} = \begin{cases} n_e - n_n & for\ \nu_{e_L} \to \bar{\nu}_{\mu_R} \\ n_e - \frac{1}{2}n_n & for\ \nu_{e_L} \to \nu_{e_R,\mu_R} \end{cases}. \tag{13}$$

As it was in the case of non-twisting magnetic field [30] the probability (7) may have a considerable value (the neutrino conversion processes and oscillations become important) if the following two conditions are valid:

1) the "amplitude of oscillations" $\sin^2 2\theta_{eff}$ is far from zero (or $\sin^2 2\theta_{eff} \sim 1$),

   and

2) the length x of the neutrinos path in the medium must be greater than the effective oscillation length L_{eff} ($x \sim$ or $> \frac{L_{eff}}{2}$).

The condition 1) is realized if $\tan 2\theta_{eff} \geq 1$, then from (9) it follows that at least one of the following two relations must be satisfied

$$\frac{\Delta m_\nu^2}{2E} A^\star - \sqrt{2} G_F n_{eff} + \dot{\phi} = 0, \quad (\tilde{\mu} B \neq 0) \tag{14}$$

$$2\tilde{\mu} B \geq \Big| \frac{\Delta m_\nu^2}{2E} A^\star - \sqrt{2} G_F n_{eff} + \dot{\phi} \Big|. \tag{15}$$

Using the definitions (see, for example, in[10]) for the oscillation length in vacuum

$$L_V = \frac{4\pi E}{\Delta m_\nu^2},$$

and the interaction oscillation length

$$L_{eff} = \frac{2\pi}{\sqrt{2} G_F n_{eff}},$$

one can write the equations (9), (10) as

$$\tan 2\theta_{eff} = L_F^{-1} \Big(\frac{A^\star}{L_V} - \frac{1}{L_{int}} + \frac{1}{L_{\dot{\phi}}} \Big)^{-1}, \tag{16}$$

$$L_{eff} = \Big[\Big(\frac{A^\star}{L_V} - \frac{1}{L_{int}} + \frac{1}{L_{\dot{\phi}}} \Big)^2 + \Big(\frac{1}{L_F} \Big)^2 \Big]^{-1/2}, \tag{17}$$

where

$$L_F = \frac{\pi}{\tilde{\mu} B}, \quad L_{\dot{\phi}} = \frac{2\pi}{\dot{\phi}}.$$

From the formula (7) we can obtain the following expressions for probability in different cases

$$P(\nu_i \to \nu_j) =$$

$$= \begin{cases} \left(\frac{L_V}{L_F A^\star}\right)^2 \sin^2\left(\frac{\pi x A^\star}{L_V}\right), & \text{for } L_F^{-1} \ll -L_{int}^{-1} + L_{\dot{\phi}}^{-1} \ll A^\star L_V^{-1}, \\ \left(\frac{L_{int}}{L_F}\right)^2 \sin^2\left(\frac{\pi x}{L_{int}}\right), & \text{for } L_F^{-1} \ll A^\star L_V^{-1} + L_{\dot{\phi}}^{-1} \ll L_{int}^{-1}, \\ \left(\frac{L_{\dot{\phi}}}{L_F}\right)^2 \sin^2\left(\frac{\pi x}{L_{\dot{\phi}}}\right), & \text{for } L_F^{-1} \ll A^\star L_V^{-1} - L_{int}^{-1} \ll L_{\dot{\phi}}^{-1}, \\ \sin^2\left(\frac{\pi x}{L_F}\right), & \text{for } A^\star L_V^{-1} - L_{int} + L_{\dot{\phi}}^{-1} = 0, \\ \to \sin^2\left(\frac{\pi x}{L_F}\right), & \text{for } L_F^{-1} \gg A^\star L_V^{-1} - L_{int}^{-1} + L_{\dot{\phi}}^{-1}. \end{cases} \tag{18}$$

The conditions (14,) (15) can be re-written as

$$\frac{A^\star}{L_V} - \frac{1}{L_{int}} + \frac{1}{L_{\dot{\phi}}} = 0, \quad L_F^{-1} \neq 0 \tag{19}$$

and

$$\frac{1}{L_F} \geq \Big|\frac{A^\star}{L_V} - \frac{1}{L_{int}} + \frac{1}{L_{\dot{\phi}}}\Big|. \tag{20}$$

Let us consider the relation (15) and suppose that the right- hand side is not equal to zero. In the case of exact equality from (15) we determine the critical strength of magnetic field [29]–[32]

$$\tilde{B}_{cr} = \left|\frac{1}{2\tilde{\mu}}\Big(\frac{\Delta m_\nu^2 A^\star}{2E} - \sqrt{2}G_F n_{eff} + \dot{\phi}\Big)\right| \tag{21}$$

that constrain the range ($B \geq \tilde{B}_{cr}$) of field strengths for which the value of $\sin^2 2\theta_{eff}$ is not small (i.e., at least is not less than $\frac{1}{2}$) for all possible values of the right-hand side term in (15).

It is also possible to express $\tilde{B}_{cr}$ in a more convenient for numerical estimation form:

$$\tilde{B}_{cr} \approx 43\Big(\frac{\mu_B}{\tilde{\mu}}\Big)\Big| - \Big(2.5\frac{n_{eff}}{10^{31}cm^{-3}}\Big) + A^\star\Big(\frac{\Delta m_\nu^2}{eV^2}\Big)\Big(\frac{MeV}{E_\nu}\Big) + 2.5\Big(\frac{1m}{L_{\dot{\phi}}}\Big)\Big|\ [Gauss]. \tag{22}$$

For the case of strong magnetic fields ($B > \tilde{B}_{cr}$), $\sin^2 2\theta_{eff} \approx 1$, we find that for large enough lengths of a neutrino ν_i pass given by $x \approx L_{eff}\frac{k}{2}, k = 1, 2, ...$ in the magnetized medium characterized by n_{eff} the probability (7) of conversion process $\nu_i \to \nu_j$ can reach the value of the order of $P(\nu_i \to \nu_j) \sim 1$.

Therefore, the initially emitted, for example, left-handed neutrino can undergo convertion to the right-handed neutrino or to the right-handed antineutrino on the path lengths $x \geq \frac{L_{eff}}{2}$.

It is obvious that these oscillation processes take place only in the presence of strong magnetic fields $B \gg \tilde{B}_{cr}$, and the oscillation length L_{eff}, as it follows from (10), is $L_{eff} \approx L_F = \pi/\tilde{\mu}B$. For $B \ll \tilde{B}_{cr}$ the influence of magnetic field is not important and oscillations (if they exist) are completely determined by the vacuum mixing angle and neutrino interaction with matter.

3 Neutrino Oscillations in Magnetic Field of Neutron Star (Cross-Boundary Effect)

Now let us consider neutrinos that are produced in the interior of a neutron star where magnetic fields of the order of 10^{13} G (or even a few orders of magnitude stronger) can exist (see, for example, [35]– [37]). For definiteness we suppose that initially ν_{eL}'s are produced in the inner layers of the neutron star and take into account the only one of the conversion processes (6),

$\nu_{e_L} \rightarrow \nu_{e_R}$, that can be induced by the magnetic field on the neutrino pass from the centre to the surface of the neutron star.

In order to determine the scale of $\tilde{B}_{cr}$ on the base of (21) and (22) we use the following values for characteristics of neutrinos and matter of the neutron star: $\mu \sim 10^{-10}\mu_B$, μ_B is the Bohr magneton, $n_{eff} \sim 10^{33}\ cm^{-3}$, $\Delta m_\nu^2 \approx 10^{-4}\ eV^2$, $\sin 2\theta = 0.1$ and $E_\nu \approx 20\ MeV$. It follows that the main contribution is given by the "matter" term and for this case

$$B_{cr} = 1.11 \times 10^{14}\ G, \tag{23}$$

(here in contrast with consideration of neutrinos from the Sun (Section 5) we do not account for a possible effect of twisting of the magnetic field). Magnetic fields just of this order may exist on the surfaces of neutron stars [36, 37].

From (10) for the effective oscillation length we get $L_{eff} \simeq 1\ cm$, that is much less than the characteristic scales of the neutron star structures (the thickness of the crust is, for instance, $L_{crust} \sim 0.1 r_0 \approx 1\ km$, r_0 is the neutron star radius).

From these estimations we can conclude that for neutrinos passing from the inner layers to the surface the conversion and oscillations effects induced by the magnetic field can be important. However, if one is dealing not with a single neutrino but with a bunch of neutrinos that are emitted in different inner points of the neutron star then the average of the x dependent term in formula (7) must be taken. Therefore the probability of ν_{eR}'s appearing in the initial bunch of ν_{e_L}'s is given by

$$\bar{P}(\nu_{e_R}) = \frac{1}{2}\sin^2 2\theta_{eff}. \tag{24}$$

It follows that the induced by strong magnetic field conversion and oscillations effects could yield in the approximate equal distribution of neutrinos between the two neutrino species ($\sin^2 2\theta_{eff} \sim 1$ if $B \gg B_{cr}$); it also means that there would be a factor of two decrease in amount of initially emitted ν_{eL}'s in the bunch.

Now let us consider the case of "not too strong magnetic field", viz., $B < B_{cr}$ along the whole neutrinos path inside the neutron star. If we exclude the possibility for the neutrinos to pass through the resonant conversion point [7, 8] determined by the Eq.(14) we then get that the neutrino bunch after travelling through the neutron star will still be composed only of the left-handed neutrinos. However, when the bunch of neutrinos escapes from the neutron star it passes through a sudden change of density of matter and enters into the nearly empty space where $n_{eff} \rightarrow 0$. Effectively it may result that the neutrinos enter and pass through the region of strong field ($B > B_{cr}$) determined on the base of Eq. (15). The neutrino conversion processes

and oscillations may thus appear due to the "cross-boundary effect" (CBE) [29, 30].

To consider the CBE we suppose that the magnetic field on the surface of the neutron star is of the order of $B \sim B_0 = 10^{12}\ G$ and that the strength of the magnetic field decreases with the distance r from the surface of the neutron star according to the law

$$B(r) = B_0 \Big(\frac{r_0}{r}\Big)^3, \tag{25}$$

where r_0 is the radius of the neutron star.

The estimation for the critical field B'_{cr} on the base of (21) for the same values of μ, Δm_ν^2, E_ν and $\sin 2\theta$ (again for definiteness the conversion of the type $\nu_{eL} \leftrightarrow \nu_{eR}$ is considered) gives that

$$B'_{cr} = 5.4 \times 10^3\ G. \tag{26}$$

From (25) it follows that the magnetic field exceeds B'_{cr} in regions characterized by

$$r \leq r_{cr} \approx 600 r_0. \tag{27}$$

Therefore, along the distances of about $600r_0$ from the neutron star the magnetic field exceeds the critical field strength B'_{cr}. From the estimation for the effective oscillation length for the magnetic field at the surface of the neutron star

$$L_{eff}(B \sim B_0) = \frac{\pi}{\tilde{\mu} B_0} \simeq 10^2 \frac{\mu_B}{\tilde{\mu}} \Big(\frac{1G}{B_0}\Big)[m] = 1\ m \tag{28}$$

it follows that the equal distribution of neutrinos between the two neutrino species (ν_{e_L} and ν_{e_R}) appears after the neutrino bunch passes through a thin layer $\Delta r \gg 1\ m$ along which the decrease of the magnetic field is still negligible: $\Delta B(\Delta r) \ll B_0$.

Thus, in the case of "not too strong field" again as it was in the case of "strong field" after the neutrino bunch has passed a distance $L > L_{eff}$ from the neutron star the equal distribution of neutrinos among the two species ν_{e_L} and ν_{e_R} appears.

Consider the case when the neutrinos on their path inside the neutron star pass through the resonant region [7, 8]. In this region the condition of Eq.(14) is valid. From (15) it follows that for any fixed strength of the magnetic field there is a layer (between the two shells with radiuses r_1 and r_2) on the neutrino path to the surface of the neutron star where effectively the "strong field" case is realized. If the distance $r_2 - r_1$ is greater than the effective oscillation length $L_{eff} \sim L_F$ then after neutrinos pass through

this *resonant region* again the equal neutrino distribution between the two neutrino species appears.

Here it is important to note that within the discussed case of the CBE the adiabatic approximation can be used. In the most general case the adiabatic condition is

$$(H_{jj}-H_{ii})\frac{\partial}{\partial r}(H_{ij}+H_{ji})-(H_{ij}+H_{ji})\frac{\partial}{\partial r}(H_{jj}-H_{ii}) \ll$$
$$2[(H_{jj}-H_{ii})^2+(H_{ij}+H_{ji})^2]^{3/2} \tag{29}$$

where H_{ij} are the elements of matrixes of eqs. (4) or (5). Using expressions for H_{ij} corresponding to the neutrino conversion $\nu_i \rightarrow \nu_j$ we reduce the adiabatic condition (29) to the form

$$\Big|\tilde{B}_{cr}\frac{\partial B}{\partial r}-B\frac{\partial \tilde{B}_{cr}}{\partial r}\Big| \ll 4\tilde{\mu}(\tilde{B}_{cr}^2+B^2)^{3/2}, \tag{30}$$

that means a slow variation of the magnetic field B and the matter density ρ ($\tilde{B}_{cr}=\tilde{B}_{cr}(n_{eff})$) with distance r. The magnetic field B is slowly varying function of r, whereas ρ undergoes a rather abrupt change from the value $\rho_s \sim 10^9\ g \times cm^{-3}$ at the surface of the neutron star to the value of nearly empty space $\rho_{vac} \rightarrow 0$. For the chosen above values of $\tilde{\mu}$, Δm_ν^2, E_ν and $\dot{\phi}=0$ we get from (30) that the adiabatic condition is valid if matter density changes from ρ_s to ρ_{vac} on the distance $\Delta r \geq \Delta r_\rho = 10\ cm \ll L_{eff}$. Therefore, even in the case of extremely abrupt change of the matter density the non-adiabatic effects can be neglected.

Now in the continuation of discussions of Refs. [28]–[30] we should also like to mention that the CBE can take place not only at the surface of the neutron star (when neutrinos escape the matter of the neutron star and start there path in the empty space where particle number densities n_e, n_n, $n_p \rightarrow 0$). The CBE can effectively appear for the Majorana neutrinos passing through inner layers of the neutron star composed of silicon, oxygen, nitrogen, carbon and helium. For these shells $n_{eff}=n_e-n_n \rightarrow 0$, that corresponds to isotopically neutral medium (see also [24]). Because of the CBE in the inner layers of the neutron star a reasonable amount of active neutrinos can be converted to the sterile (non interacting with matter) neutrinos that may cause changes in the process of neutron stars cooling.

In the next Section we shell study the CBE at the surface of the neutron star using a model of the neutron star structure provided by a realistic equation of state for the matter of neutron star.

4 Cross-Boundary Effect for Realistic Neutron Star Structure

Let us consider the CBE at the surface of the neutron star using a realistic model of the star structure to account for change of the matter density with distance from the centre of the star.

Neutron star structure is calculated (see, for example, [36]) assuming that the equation of state for neutron star matter, $P = P(\rho)$ (P is the pressure, ρ is the mass density) at $\rho \geq 2 \times 10^{14}(g \times cm^{-3})$ is that of three-nucleon interaction (TNI) model [38]. For ρ within the interval $4.3 \times 10^{11}(g \times cm^{-3}) \leq \rho \leq 2 \times 10^{14}(g \times cm^{-3})$ the Baym-Bethe-Pethieck (BBP) equation of state is used. At $\rho < 4.3 \times 10^{11}(g \times cm^{-3})$ we use the Baym-Pethick- Sutherland (BPS) equation of state [36, 39].

For description of the non-rotating star composed of cold matter one have to integrate the general-relativistic equation of hydrostatic balance, the Tolmen-Oppenheimer-Volkoff (TOV) equation [36]

$$\frac{dP}{dr} = -\frac{G(\rho + P)(m(r) + 4\pi r^3 P)}{r^2(1 - 2Gm(r)/r)}, \tag{31}$$

$$m(r) = \int_0^{(r)} \rho(r') d^3 r', \tag{32}$$

where $m(r)$ is the mass of the star, r is radial coordinate ($r = 0$ for the centre of the neutron star), G is the gravitational constant.

In equation (31) the preasure P is defined as a function of the density ρ by equation of state. For outside of neutron star we use the BPS equation of state. This model implay that matter is composed of free nuclei, electrons and neutrons. The Coulomb interaction energy is also accounted and the equation of state can be represented as a system (see, for example, [36]):

$$\begin{cases} \rho = \varepsilon = n_e M(A, Z)/Z + \varepsilon_e + \varepsilon_L, \\ P = P_e + P_L, \end{cases} \tag{33}$$

Here ε is the total energy (per unit volume), n_e is the electron density, $M(A, Z)$ is the energy of a nucleus (A, Z), ε_e is the energy of electrons without the energy of the Coulomb interaction, ε_L is the Coulomb interection between electrons and electrons with nuclei, P_e is the partial pressure of the electrons and $P_L = \frac{1}{3}\varepsilon_L$.

The values $n_e, \varepsilon_L, P_e, \varepsilon_e$ are represented as functions of parameter $X_e = p_F^e/m$ (p_F^e is the electron Fermi momentum) [36]:

$$n_e = \frac{1}{3\pi^2\lambda_e^3} X_e^3 \tag{34}$$

($\lambda_e = \frac{1}{m_e}$ is the electron compton wavelength),

$$\varepsilon_L = -1.444 Z^{2/3} e^2 n_e^{4/3}, \tag{35}$$

$$P_e = \frac{m_e}{\lambda_e^3} \Phi(X_e), \tag{36}$$

where $\Phi(x) = \frac{1}{8\pi^2}\{x(1+x^2)^{1/2}(1+2x^2) - \ln[x + (1+(1+x^2)^{1/2}]\}$,

$$\varepsilon_e = \frac{m_n}{\lambda^3} \chi(X_e), \tag{37}$$

where $\chi(x) = \frac{1}{8\pi^2}\{x(1+x^2)^{1/2}(1+2x^2) - \ln[x + (1+x^2)^{1/2}]$.

We assume that the density of the free neutrons is equal to zero because it is below the density of the neutron drip [36]. The values of A and Z used in the sistem (33) minimize the energy ε that corresponds to the equilibrium nucleus.

For definiteness we again consider the case of oscillations among the Majorana neutrinos ($\nu_{e_L} \leftrightarrow \overline{\nu}_{\mu_R}$). From (13) whithin the discussed model of the neutron star matter we have

$$n_{eff} = n_e - \Big(\frac{A}{Z} - 1\Big) n_e. \tag{38}$$

Using eqs. (33)–(37) we perform a computer calculations of n_{eff} as a function the distance from the centre of the neutron star and then determine B_{cr} (see (21)) for the different values of r.

It is interesting to compare the calculated value of the critical field $B_{cr}(r)$ and the neutron star magnetic field $B(r)$ for different distances r from the centre of the neutron star. We suppose that the neutron star magnetic field for rather large distances from the surface ($r \geq r_0 \approx 10\ km$) can be approximated by eq. (25). However, in the close to the star surface layers the magnetic field may exhibit the more complicated behavior with variation of r. It is possible to suppose that the field decrease with the distance from the surface of the star ($r = r_0$) not faster than it follows from the law $B \sim \rho^{2/3}$ (the field frozen in the matter).

Using these suggestions on the profile of the magnetic field of the star we plot (Fig. 1) the dependence of the critical field $B_{cr}(r)$ (solid line) and the neutron star magnetic field $B(r)$ on the distance r for the close to the star surface layers ($r \sim r_0$). It is supposed that $B(r = r_0) \approx 10^{14} G$. The solid line with dots corresponds to the case when the star magnetic field $B \sim \rho^{2/3}$.

The dashed line shows the behavior of the field for the case when the field profile is given by $B(r) \sim 1/r^3$. is

Fig. 2 shows the dependence of $B_{cr}(r)$ (solid line) and the neutron star magnetic field $B(r) \sim 1/r^3$ (solid line with dots) on r for remote distances $(r \geq r_0)$. For large distances $(r \gg r_0)$ $B_{cr}(r)$ nearly equal to its vacuum value

$$B_{cr} \approx \frac{\Delta m_\nu^2 A^\star}{4\tilde{\mu}E}.$$

From these figures it follows that nearly for the whole space from the surface of the neutron star ($r_1 = 10.32\, km$) up to the distances $r_2 \sim 10^3\, r_0$ the magnetic field $B(r)$ exceeds the critical field $B_{cr}(r)$. Taking into account that for the field $B \sim 10^{14} G$ the effective oscillation length L_{eff} determined by eq. (10) is of the order of $\sim 1\ cm$ we conclude that the magnetic field induced neutrino oscillation effects can be important for the space characterized by $r_1 \leq r \leq 10^3\ r_0$.

On Fig. 3 we plot the averaged probability $P_{av} = \overline{P}(\overline{\nu}_{\mu_R})$ (similar to one of eq. (24)) that determines the amount of neutrinos $\overline{\nu}_{\mu_R}$ in the initial bunch of neutrinos ν_{e_L} as a function of r. For each individual neutrino (initially emited as ν_{e_L}) the non-averaged probabilty $P(\overline{\nu}_{\mu_R})$ to detect the neutrino in

the state $\overline{\nu}_{\mu_R}$ oscillate with the change of r around the average value $\overline{P}(\overline{\nu}_{\mu_R})$ with the amplitude determined by $B(r)/B_{cr}(r)$.

of the probability variation

$P(\overline{\nu_{\mu_R}})$ in the narrow

5 Supernova Reheating Problem, Neutrino Oscillations in Galactic and Twisting Solar Magnetic Field

The effects discussed above of suppression of amount of electron neutrinos (or other active neutrinos) induced by strong magnetic fields may have sufficient consequences on the reheating phase of a Type II supernova that can be used for getting constraints on the value $\tilde{\mu}B$. Let us suppose that the magnetic field induced neutrino oscillations do not destroy the proposed model [16] of about 60 % increase in the supernova explosion energy. If the magnetic field $B \sim 10^{14}\ G$ exists at the radius of $r_0 = 45\ km$ from the centre of the hot proto neutron star (the matter density in this region is $\rho \sim 10^{12}\ g/cm^3$) and decreases with distance according to (25) then on the distances $r \sim 160\ km$ from the centre the magnetic field is $\sim 0.6 \times 10^{13}\ G$. This field is of the order of the B_{cr} determined by (21) for the density $\rho \sim 6 \times 10^8\ g/cm^3$ and the magnetic moment $\tilde{\mu} \sim 10^{-10}\mu_B$. For this case the probability of finding, for example, sterile ν_{eR}'s among the initially emitted ν_{eL}'s is $\bar{P}_{\nu_{eL}\to\nu_{eR}} = 0.25$ (the effective length (10) for this effect is $L_{eff} \sim\ 10\ cm$). Therefore, in order to avoid the loss of a substantial amount of energy that will escape from the region behind the shock together with the sterile neutrinos, one has to constrain the magnetic moment on the level of $\tilde{\mu} \leq 10^{-11}\mu_B$.

We should like to point out the importance of the resonance enhancement [7, 8] of neutrino conversion and oscillations effect in magnetic fields that may substantially change the energetics of the shock and also give a stringent constraints on the value of $\tilde{\mu}B$.

It is also interesting to consider the neutrino conversion and oscillations induced by the interstellar galactic magnetic fields that are of the order $B_G \sim 10^{-6}\ G$. The critical field estimated on the base of eqs. (21) for ultra high energy neutrinos ($E \geq 10^{17}\ eV$) are $\leq 10^{-6}\ G$. Taking into account the estimation for the effective oscillation length $L_{eff}(B \sim B_G) = 10^{20}\ cm$, that is much less than the radius of the galaxy ($R_G \approx 3 \times 10^{22}\ cm$) we conclude that in this case the effect of neutrino conversion and oscillations in "strong magnetic field" can be presented.

Now let us consider the possibility of the twisting magnetic field induced

neutrino conversion and oscillations (for definiteness we chose the process $\nu_{e_L} \to \nu_{e_R}$) in the convective zone of the Sun. First of all we estimate the critical field strength, using eqs. (21) with the following values for characteristics of neutrinos and matter: $\Delta m_\nu^2 = 10^{-4}\ eV^2$, $\sin 2\theta = 0.1$, $E_\nu = 20\ MeV$, $n_{eff} \sim n_e \approx 10^{23}\ cm^{-3}$. For the characteristic of the variation of the field in the convective zone along the neutrinos path we suppose that $\dot\phi > 0$ and use the estimation of Refs.[24, 33]: $L_{\dot\phi} \sim 0.1 R_\odot \approx 7\times 10^7\ m$, $R_\odot = 7\times 10^8\ m$ is the solar radius. Substituting these values to three terms of eq. (21) we get

$$\tilde B_{cr} \approx \Big(\frac{\mu_B}{\tilde\mu}\Big)\Big| - 10^{-6} - 5\times 10^{-7} + 1.43\times 10^{-6}\Big|\ G = 7\times 10^{-8}\Big(\frac{\mu_B}{\tilde\mu}\Big)\ G. \quad (39)$$

From this it is obvious that the account for the twisting of magnetic field reduce the value of critical field $\tilde B_{cr}$ to the order of 5% of the value B_{cr} that corresponds to the case of non-twisting field.

It is supposed that the typical value of magnetic fields in the convective zone is of the order of $B_{con} \sim 10^5\ G$. From (25) it follows that B_{con} exceeds the field $\tilde B_{cr}$ if the neutrino magnetic moment is greater then $\tilde\mu \geq 10^{-12}\mu_B$.

According to the second condition for the magnetic field induced neutrino conversion and oscillations become important the effective oscillation length L_{eff} have to be of the order or less then the depth of the convective zone $L_{eff} \leq \frac{1}{2} L_{cz}$. This last condition holds if $\tilde\mu \sim 10^{-11}\mu_B$ for the magnetic fields in the convective zone $B \sim 10^5\ G$.

We can conclude that the account for the variation (twisting) of the magnetic field along the neutrino path in the solar convective zone relaxes the critical field strength $\tilde B_{cr}$ to the values which can be relevant for the stimulation of a visible neutrino conversion and oscillations if the neutrino magnetic moment is of the order of $\tilde\mu \sim 10^{-11}\mu_B$.

6 Summary

The Dirac and Majorana neutrino conversion and oscillations between the two neutrino species induced by the magnetic field is considered. We introduce the critical magnetic field strength $\tilde B_{cr}(\Delta m_\nu^2, \theta, n_{eff}, E, \tilde\mu, \dot\phi)$ that determines the range of fields ($B \geq \tilde B_{cr}$) for which the magnetic field induced neutrino conversion and oscillations become important. This criterion is valid in the case of resonant and non-resonant amplification of neutrino conversion and oscillations in magnetic fields.

The criterion is used in the study of the neutrino conversion and oscillations in magnetic fields of neutron star, supernova, the Sun and interstellar galactic media. The possible conversion of one half of neutrinos from active

into sterile specie on the neutrino bunch crossing the surface (the “cross-boundary effect”) of the neutron star is predicted and discussed in some details.

7 Acnowledgments

The authors are thankful to A.Dar, B.Mayer and S.Petcov for useful discussions. One of the authors (A.I.S.) should also like to thank G.Bellittini and M.Greco and all the organizers of the “Recontres de Physique de La Thuile” for their kind hospitality.

This work was supported in part by the Interregional Centre for Advanced Studies.

References

[1] L.Wolfenstein, Phys.Rev. D17, 2369 (1978).

[2] S.P.Mikheyev, A.Yu.Smirnov, Sov.J.Nucl.Phys. 42, 913 (1985).

[3] S.P.Mikheev, A.Yu.Smirnov, Nuovo Cimento C9, 17 ()1986.

[4] A.Cisneros, Astrophys.Space Sci. 10, 87 (1981).

[5] M.B.Voloshin, M.I.Vysotsky, L.B.Okun, Sov.J. Nucl. Phys. 44, 440 (1986).

[6] L.B.Okun, M.B.Voloshin, M.I.Vysotsky, Sov.J.Nucl.Phys. 44, 440 (1986).

[7] E.Kh.Akhmedov, Phys.Lett. B213, 64 (1988).

[8] C.-S.Lim, W.Marciano, Phys.Rev. D37, 1368 (1988).

[9] V.Berezinsky, On Astrophysical Solution to the Solar Neutrino Problem, LNGS-94/101.

[10] J.N.Bahcall: Neutrino Astrophysics, Cambridge University Press, 1989.

[11] J.Pulido, Phys.Rep. 211, 167 (1992).

[12] X.Shi, D.N.Schramm, R.Rosner, D.S.Dearborn, Comm. Nucl. Part. Phys. 21, 151 (1993).

[13] B.P.Pal, Int.J.Mod.Phys. A7, 5387 (1992).

[14] M.B.Voloshin, JETP Lett. 47, 501 (1988).

[15] X.Shi, G.Sigl, Phys.Lett. B323, 360 (1994).

[16] G.M.Fuller, R.Mayle, B.S.Meyer, J.R.Wilson, ApJ 389, 517 (1992).

[17] A.Yu.Smirnov, D.N.Spergel, J.N.Bahcall, Phys.Rev D49,1389 (1994).

[18] G.Raffelt, G.Sigl, Astopart.Phys. 1, 165 (1993).

[19] J.T.Peltoniemi, Astron.Astrophys. 254, 121 (1992).

[20] H.Athar, J.T.Peltoniemi, A.Yu.Smirnov, Preprint of the International Centre for Theoretical Physics (Trieste, Italy), IC/94/399 (1994).

[21] B.Pontecorvo, Zh.Eksp.Theor.Fiz. 34, 247 (1958).

[22] V.N.Gribov, B.Pontecorvo, Phys.Lett. B28, 493 (1969).

[23] S.M.Bilenky, B.Pontecorvo, Phys.Lett. B61, 248 (1976).

[24] E.Kh.Akhmedov, S.T.Petcov, A.Yu.Smirnov, Phys.Rev. D48, 2167 (1993).

[25] A.V.Averin, A.V.Borisov, A.I.Studenikin, Phys.Lett. B231, 280 (1989).

[26] G.G.Likhachev, A.I.Studenikin, Sov.J.Nucl.Phys. 55, 150 (1992).

[27] G.Raffelt, Phys.Rep. 198, 1 (1990).

[28] A.I.Studenikin, "Neutrino Conversion and Oscillations in Strong Magnetic Fields", talk given at the Summer Institute on Nuclear Physics and Astrophysics (June 1994), Laboratori Nazionale del Gran Sasso, Italy, (unpublished).

[29] G.G.Likhachev, A.I.Studenikin, in Proc. of Ist Int. Conf. on Phenomenology of Unification from Present to Future, ed. by G.Diambrini-Palazzi, L.Zanello, G.Martinelli, World Scientific, Singapore, 1994.

[30] G.G.Likhachev, A.I.Studenikin, Preprint of the International Centre for Theoretical Physics (Trieste, Italy), IC/94/170, 1994, 8p.

[31] G.G.Likhachev,A.I.Studenikin, Grav.& Cosm. 1, 22 (1995).

[32] G.G.Likhachev, A.I.Studenikin, Hadronic Journal 18, 1 (1995).

[33] A.Yu.Smirnov, Phys.Lett.B260, 161 (1991).

[34] J.Vidal, J.Wudka, Phys.Lett. B249, 473 (1990).

[35] J.P.Landstreet, Phys.Rev. 153, 1372 (1967).

[36] S.L.Shapiro, S.A.Teukolsky: Black Holes, White Dwarfts and Neutron Stars, Wiley, New York, 1983.

[37] V.M.Lipunov: Astrophysics of Neutron Stars, Nauka, Moscow, 1987.

[38] B.Friedman, V.Pandharipande, Nucl. Phys. A, 361, 502 (1981).

[39] G.Baym, C.Pethick, P. Sutherland, ApJ 170, 299 (1971).